

NNK and $\Lambda\Lambda K$ Amplitudes in Chiral Perturbation Theory

M.J. Savage^{a*} and R.P. Springer^b

^aDepartment of Physics and Astronomy, Box 351560,
University of Washington, Seattle, WA 98195-1560, USA

^bDepartment of Physics, Duke University,
Durham, NC 27708, USA

We explore the S-wave and P-wave amplitudes for NNK and $\Lambda\Lambda K$ interactions using chiral perturbation theory to $\mathcal{O}(m_s \log m_s)$. In contrast to the large corrections found in P-wave amplitudes in the nonleptonic decay of octet baryons, the NNK amplitudes receive only small corrections to their tree-level values. The uncertainty in the $\Lambda\Lambda K$ amplitudes is found to be large.

1. Introduction

The motivation for studying the NNK and $\Lambda\Lambda K$ amplitudes is two-fold. First, one would like to determine if the problems chiral perturbation theory (χPT) has in describing the P-wave amplitudes for the nonleptonic decay of the octet-baryons, $B \rightarrow B'\pi$, at leading order represent a failure of χPT or just an accident[1]. Second, one wishes to accurately determine these amplitudes to better understand the decay of hypernuclei.

The on-shell NNK and $\Lambda\Lambda K$ interactions are not directly observable. In order to make theoretical predictions for processes involving these interactions the symmetries of QCD are used to relate them to other observables such as $B \rightarrow B'\pi$. χPT provides a natural framework to do this in the limit of exact SU(3) and also allows one to systematically include SU(3) breaking effects arising from the mass difference between the strange and the up and down quarks. Local counterterms are analytic functions of the light quark masses while loop graphs involving the lightest octet of pseudo-Goldstone bosons give rise to contributions that are non-analytic in the light quark masses. These non-analytic contributions are unique and dominate local counterterms in the chiral limit.

χPT does poorly in describing the P-wave amplitudes of $B \rightarrow B\pi$ at leading order. Table 1 shows the experimental and theoretical amplitudes both at loop and tree-level. The nonleptonic weak couplings are fit to the experimentally measured S-wave amplitudes in $B \rightarrow B\pi$, and together with the experimentally measured axial matrix elements give rise to theoretical predictions for the P-wave amplitudes[1]. It was suggested in [1] that the failure of leading order χPT to reproduce the P-wave amplitudes does not represent a failure of χPT but rather is an accident. There are two pole graphs that contribute to

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Table 1
S-wave and P-wave amplitudes for the nonleptonic decay of octet baryons

	S-wave			P-wave		
	expt	tree (fit)	loop (fit)	expt	tree	loop
$\Sigma^+ \rightarrow n\pi^+$	0.06	0.0	0.00 ± 0.02	1.81	-0.18	0.82
$\Sigma^+ \rightarrow p\pi^0$	-1.43	-1.4 ± 0.1	-1.3 ± 0.1	1.17	-0.35	0.36
$\Sigma^- \rightarrow n\pi^-$	1.88	2.0 ± 0.2	1.9 ± 0.2	-0.06	0.32	0.34
$\Lambda \rightarrow p\pi^-$	1.43	1.5 ± 0.2	1.4 ± 0.2	0.52	-0.56	-0.52
$\Lambda \rightarrow n\pi^0$	-1.04	-1.1 ± 0.1	-1.0 ± 0.1	-0.39	0.40	0.38
$\Xi^- \rightarrow \Lambda\pi^-$	-1.98	-1.9 ± 0.1	-2.0 ± 0.2	0.48	-0.13	0.35
$\Xi^0 \rightarrow \Lambda\pi^0$	1.52	1.4 ± 0.1	1.4 ± 0.1	-0.33	0.18	-0.24

the P-wave amplitudes at leading order and for the measured value of couplings there is substantial cancellation between them. Understanding this cancellation is beyond χPT , but it is suggestive that it is accidental in this system. In addition, one sees that the non-analytic corrections $\mathcal{O}(m_s \log m_s)$, bring the amplitudes into better, but still poor agreement. The uncertainties in the theoretical predictions of the P-wave amplitudes are currently under investigation by one of us[2].

2. The NNK Amplitudes

There are three vertices that occur in $\Delta s = 1$ weak nonleptonic interactions involving nucleons and kaons; $p\bar{p}K^0$, $n\bar{p}K^+$, and $n\bar{n}K^0$. Isospin symmetry relates these amplitudes

$$\mathcal{A}^{(L)}(nnK) - \mathcal{A}^{(L)}(ppK) = \mathcal{A}^{(L)}(npK) \quad , \quad (1)$$

where $L = 0$ (S-wave) or $L = 1$ (P-wave). The amplitudes can be written in terms of contributions at a given order in perturbation theory

$$\mathcal{A}^{(L)} = \mathcal{A}_0^{(L)} + \mathcal{A}_1^{(L)} + \dots \quad , \quad (2)$$

where the subscript denotes the order in chiral perturbation theory and the dots denote higher orders.

The $\Delta s = 1$ weak interactions of the pseudo-Goldstone bosons and the lowest lying baryons are described, assuming octet dominance, by the Lagrange density

$$\begin{aligned} \mathcal{L} = & G_F m_\pi^2 f_\pi \left(h_D \text{Tr} \bar{B}_v \{ \xi^\dagger h \xi, B_v \} + h_F \text{Tr} \bar{B}_v [\xi^\dagger h \xi, B_v] + h_C \bar{T}_v^\mu (\xi^\dagger h \xi) T_{v\mu} \right. \\ & \left. + \frac{h_\pi}{8} \text{Tr} \left(h \partial_\mu \Sigma \partial^\mu \Sigma^\dagger \right) + \dots \right) \quad , \end{aligned} \quad (3)$$

where

$$h = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad , \quad (4)$$

and the constants f_π , h_D , h_F , h_π and h_C are determined experimentally at a given order in the expansion. The octet baryon field is denoted by B_v and the decuplet field by T_v^μ [3].

2.1. S-Wave Amplitudes

At tree level the S-wave amplitudes appear directly from the first and second terms in the weak Lagrangian of Eq. 3 ,

$$\mathcal{A}_0^{(S)}(p\bar{p}K^0) = h_F - h_D \quad , \quad \mathcal{A}_0^{(S)}(p\bar{n}K^+) = h_F + h_D \quad , \quad \mathcal{A}_0^{(S)}(n\bar{n}K^0) = 2h_F \quad (5)$$

The measured S-wave amplitudes for $B \rightarrow B'\pi$ at tree level lead to $h_D = -0.58$ and $h_F = 1.40$ [1]. Therefore, the tree level S-wave amplitudes for NNK are 2.0, 0.8, and 2.8, respectively. At one-loop the computation involves several loop graphs as discussed in [4] and the effects of the $\mathcal{O}(m_s \log m_s)$ breaking are shown in Table 2. One finds that the central value of each amplitude is suppressed compared to tree-level by $\sim 30\%$. The uncertainty for each amplitude is found to be relatively small. The impact of these results has been explored in [5], and presented by A. Ramos at this workshop. They find that the ratio of neutron induced to proton induced decays of Λ -hypernuclei and the total decay rate are modified at the 10% – 15% level by the reduced amplitudes shown in Table 2.

2.2. P-Wave Amplitudes

The tree level P-wave amplitudes come directly from pole graphs involving one weak vertex from Eq. 3 and one strong vertex

$$\begin{aligned} \frac{\mathcal{A}_0^{(P)}(p\bar{p}K^0)}{\Lambda_\chi} &= -\frac{(D-F)(h_D-h_F)}{m_N-m_\Sigma} \\ \frac{\mathcal{A}_0^{(P)}(p\bar{n}K^+)}{\Lambda_\chi} &= -\frac{1}{6}\frac{(D+3F)(h_D+3h_F)}{m_N-m_\Lambda} + \frac{1}{2}\frac{(D-F)(h_D-h_F)}{m_N-m_\Sigma} \\ \frac{\mathcal{A}_0^{(P)}(n\bar{n}K^0)}{\Lambda_\chi} &= -\frac{1}{6}\frac{(D+3F)(h_D+3h_F)}{m_N-m_\Lambda} - \frac{1}{2}\frac{(D-F)(h_D-h_F)}{m_N-m_\Sigma} \end{aligned} \quad (6)$$

The numerical values for these amplitudes are found by using tree-level parameters $h_D = -0.58$, $h_F = 1.40$, $D = 0.8$, and $F = 0.5$ [1,3]. This yields tree level P-amplitudes of -2.4 , 9.1 , and 6.7 ($\times \Lambda_\chi/1 \text{ GeV}$), respectively. It is important to notice that there is only one pole graph contributing to each amplitude. Therefore the cancellations that were found to be problematic in $B \rightarrow B'\pi$ do not occur for the NNK amplitudes. These amplitudes are found to be of natural size in contrast to those for $B \rightarrow B'\pi$. Loop contributions to these amplitudes were computed to $\mathcal{O}(m_s \log m_s)$ and found to be moderately small. Measurement of the NNK amplitudes would provide valuable insight into the applicability of χPT for these processes. If there was agreement then this would suggest that the problem with the P-wave amplitudes in $B \rightarrow B'\pi$ is merely an accident and is not a failure of χPT . If the agreement is poor then this suggests that we really do not have a good understanding of these processes.

3. The $\Lambda\Lambda K$ Amplitudes

The S-wave and P-wave amplitudes for $\Lambda\Lambda K$ are determined in the same way as the NNK amplitudes[6]. At tree-level we have

$$\begin{aligned} \mathcal{A}_0^{(S)}(\Lambda\Lambda K^0) &= -h_D \\ \frac{\mathcal{A}_0^{(P)}(\Lambda\Lambda K^0)}{\Lambda_\chi} &= \frac{1}{6}\frac{(D-3F)(3h_F-h_D)}{m_\Lambda-m_\Xi} - \frac{1}{6}\frac{(D+3F)(h_D+3h_F)}{m_\Lambda-m_N} \end{aligned} \quad , \quad (7)$$

and notice that there are contributions from two pole graphs as for $B \rightarrow B'\pi$. The loop contributions of the form $\mathcal{O}(m_s \log m_s)$ are found to move the central value of the amplitudes only slightly from their tree-level values, but the uncertainties become very large. It really only makes sense to quote a range for both the S-wave and P-wave amplitudes, as shown in Table 2. In making predictions for the decay of doubly- Λ hypernuclei these large uncertainties must be considered.

Table 2
S-wave and P-wave amplitudes for NNK and $\Lambda\Lambda K$

	S-wave		P-wave	
	tree	loop	tree	loop
nnK	2.8	1.9 ± 0.4	6.7	6 ± 1
ppK	2.0	1.5 ± 0.1	-2.4	-1.7 ± 0.2
npK	0.8	0.4 ± 0.1	9.1	7 ± 1
$\Lambda\Lambda K$	0.6	$-0.5 \lesssim \mathcal{A} \lesssim 1.1$	-5.2	$-9 \lesssim \mathcal{A} \lesssim -0.5$

4. Conclusions

We have computed the $\mathcal{O}(m_s \log m_s)$ corrections to the S-wave and P-wave amplitudes for the NNK and $\Lambda\Lambda K$ interaction. The NNK amplitudes are not sizably modified from their tree-level values, in contrast to the P-wave amplitudes for $B \rightarrow B'\pi$. Measurement of these amplitudes in hypernuclear decay would provide a valuable test of chiral perturbation theory. The $\Lambda\Lambda K$ amplitudes are poorly constrained at loop-level and this large uncertainty must be included in predictions for the decay of doubly- Λ hypernuclei. The role of higher order counterterms, while not explicitly discussed here, has been estimated in the theoretical uncertainty assigned in the fitting procedure, and discussion can be found in [4,6].

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